Solution by G. B. M. ZERR, A. M., Ph.D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let $\delta=12^\circ\ 40'$, φ=latitude of observation, $a=16^\circ\ 40'$, $a'=40^\circ\ 20'$, $\mu=51^\circ=3$ hours, 24 minutes, h=hour angle.

$$\text{Then } \cos h=\frac{\sin a-\sin\varphi\sin\delta}{\cos\varphi\cos\delta}=x,\quad \cos(h-\mu)=\frac{\sin a'-\sin\varphi\sin\delta}{\cos\varphi\cos\delta}=y.$$

Eliminating h, $\sin^2\mu=x^2-2xy\cos\mu+y^2$. Substituting values of x and y,

$$\cos^2\varphi\cos^2\delta\sin^2\mu=2\sin^2\varphi\sin^2 d(1-\cos\mu)+\sin^2 a+\sin^2 a'-2\sin a\sin a'\cos\mu$$
$$-2\sin\varphi\sin\delta(1-\cos\mu)(\sin a+\sin a').$$

$$\therefore\ \sin^2\varphi[\sin^2\mu+\sin^2\delta(1-\cos\mu)^2]-2\sin\varphi\sin\delta(1-\cos\mu)(\sin a+\sin a')$$
$$=\cos^2\delta\sin^2\mu-\sin^2 a-\sin^2 a'+2\sin a\sin a'\cos\mu.$$

Let $\sin^2\mu+\sin^2\delta(1-\cos\mu)^2=A=.610569$.
$\sin\delta(1-\cos\mu)(\sin a+\sin a')=B=.075921$.
$\cos^2\delta\sin^2\mu-\sin^2 a-\sin^2 a'+2\sin a\sin a'\cos\mu=C=.307394$.

$\therefore\ A\sin^2\varphi-2B\sin\varphi=C$.

$$\therefore\ \sin\varphi=\frac{B\pm\sqrt{(AC+B^2)}}{A}=.844702,\text{ or } -.596013.\quad \varphi=57^\circ\ 38'\ 37''.$$

PROBLEMS FOR SOLUTION.

ARITHMETIC.

166. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, Ohio.

If I sell one of my farms for $\$A$,=\$4500, and the other for $\$B$,=\$1800, I will gain $p\%$, =5%, on cost of both; but if I sell the dearer farm for $\$C$,=\$4000, and the other at cost, I will lose $p\%$,=5%. Find the cost of each farm.

AVERAGE AND PROBABILITY.

138. Proposed by G. B. M. ZERR, A. M., Ph. D, Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Find the average area of (1) triangle, (2) quadrilateral, (3) pentagon, (4) hexagon, formed by taking (1) three, (2) four, (3) five, (4) six random points on the circumference of a given circle radius a.

139. Proposed by L. C. WALKER, A. M., Graduate Student, Leland Stanford Jr. University, Cal.

Four points are taken at random on the surface of a given sphere; find the average volume of the tetrahedron formed by the planes passing through the points taken three and three.